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Identification of Continuous-Time, Linear, and Nonlinear Models of an Electromechanical Actuator

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A pragmatic approach to the identification of type 1, continuous-time, linear, and nonlinear models of dynamic systems is presented. This type of model is common for electromechanical actuators, and the approach is presented using the identification of models for a particular electromechanical actuator. The experimental setup, model identification, parameter estimation, and model verification are discussed. The approach utilizes state-variable filters and least-squares in the parameter estimation procedure. The rationale for the estimation of parameters of a continuous-time model is given by contrasting it with the estimation of parameters of a discrete-time model, which is far more common and simple. Rationale is also given for the use of the simple least-squares algorithm rather than more complicated methods such as instrumental-variable methods, which may be more applicable in theory. The application of the approach to the identification of both linear and nonlinear models of the electromechanical actuator resulted in acceptable models, as determined through verification procedures using several sets of experimental data in simulation. Although it would be impossible to completely validate the approach for the nonlinear model, the results for the specific case to which it was applied are encouraging.

Nomenclature

B = lumped viscous friction parameter
b, c = coefficients (parameters) of the logarithmic function of velocity for the friction force, $F_{\text{fric}} = b\,\ell n(|\dot{x}|) + c$
C = lumped static coefficient of friction
$\boldsymbol{E}$ = $m \times 1$ vector of errors (residuals) in a regression model
F = applied force
F_f, $\boldsymbol{F}_f$ = applied force after filtration, and $m \times 1$ observation vector in the regression model composed of the applied forces after filtration
F_{fric} = nonviscous friction force
$G(s)$ = continuous-time transfer function of a system
$G_0(s)$ = continuous-time transfer function of a type 0 system
$G_1(s)$ = continuous-time transfer function of a type 1 system
$H(s)$ = continuous-time transfer function of a filter
M = lumped mass parameter
m = number of discrete sample times for a data set
n = number of parameters to be estimated in a model
s = complex variable, $\sigma + j\omega$, used in the Laplace transform, alternatively it may be thought of as the differential operator, d/dt
x = linear position
x_f = linear position after filtration
$\boldsymbol{Y}$ = $m \times 1$ observation vector in a regression model
u = $n \times 1$ vector of unknowns (parameters) in a regression model
Φ = $m \times n$ matrix of regressors in a regression model
Φ^* = pseudoinverse of Φ

Introduction

THE use of electromechanical actuation is becoming increasingly popular in the aerospace industry as more importance is placed on maintainability. This technology is being used in many situations where hydraulics were employed almost exclusively in the past. Electromechanical actuators (EMAs) are being used in the actuation of flight critical control surfaces and in thrust vector control. A good understanding of the dynamic properties of these actuators is critical in their successful application.

The identification of models for dynamic systems is an important part of system design and analysis. Dynamic models for actuators are used in control design, performance analysis, simulation, and in the analysis of their integration into more complex systems. A good dynamic model of an actuator will assist in the analysis of the actuator's impact on power generation requirements, dynamic performance of vehicles, and forces generated in backdriving situations, such as during startup and shutdown of unthrottled rocket engines that are actuated by an EMA in a thrust vector control application.

Parameter estimation is a significant problem within the larger problem of system identification. After a parametric model has been identified, the problem of determining the parameters for the model still remains. Parameter estimation may be described as the determination of the coefficients for the differential equation(s) describing the system. The coefficients of a dynamic model are often determined by applying physical principles and performing calculations using manufacturer's data and known properties of the components. This is an important part of the initial stages of the design of dynamic systems. However, many of the coefficients are difficult to accurately predict in this manner, and some of the dynamic effects are often overlooked. For example, the friction effects in mechanical systems are very difficult to predict and are best determined experimentally. The experimental determination of the parameters is defined as parameter estimation.

It is important that the models identified and estimated for a dynamic system be as useful as possible. The problem addressed in this paper is the estimation of continuous-time, linear, and nonlinear models for a dynamic system. There are many texts that deal with parameter estimation methods.[1–3] The most common methods utilize linear-regression models and least-squares to identify the parameters of difference equations for discrete-time, linear models. Least-squares is utilized in the parameter estimation procedures described here. However, it is used for a continuous-time model. The quantity of

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literature dealing with parameter estimation for continuous-time models is minute in comparison with that dealing with discrete-time models. Survey papers on the subject are given by Young[4] and Unbehauen and Rao.[5] Despite the fact that most conceptual control design is still done with continuous-time models and that a good intuitive understanding of a system is often obscured by a discrete-time model, the application of continuous-time parameter estimation procedures is very limited.

EMA Test Stand

The system identification was performed using data collected from experiments performed on an EMA in the University of Alabama Electromechanical Actuation Test Facility. The general configuration of the test stand is shown in Fig. 1. It was designed to produce large dynamic loads on a linear actuator, under force or position control. It was also designed to accept either rigidly mounted linear actuators, as shown in Fig. 1, or self-contained actuators with pivoting end connectors, like clevis mounts. The purpose of the carriage with a rigidly mounted actuator is to absorb the reaction torque of the nut resulting from axial loads. The carriage can also be used to support one end of a self-contained actuator for testing.

The electromechanical-hydraulic drive train used in the experiments is shown in Fig. 2. This figure shows the hydraulic loading system coupled to the EMA. The axial load generated by the hydraulic system is transformed to torsion in the roller nut, and is transferred to the end beam of the test stand through the bearings on the roller screw. The load cell directly measures the force applied to the nut cage and nut, eliminating the dynamics of the loading system and the test stand from the force measurement. Both the load cell and the extension pipe are hollow, allowing the roller screw to extend through them. This is necessary for the load cell to be connected directly to the nut cage. The linear variable differential transformer (LVDT) measures the displacement of the nut relative to the end beam and bearings.

If it is assumed that the test stand structure is stiff, all of the energy of the measured axial load is either converted to mechanical energy in the EMA actuator system, dissipated in the EMA actuator system, or leaves the system through a torque applied to the motor. This is an important requirement for accurate dynamic analysis of the actuator system using the measured load.

The EMA actuator system includes the nut cage, nut, roller screw, radial/thrust bearings, gear reduction, and the motor.

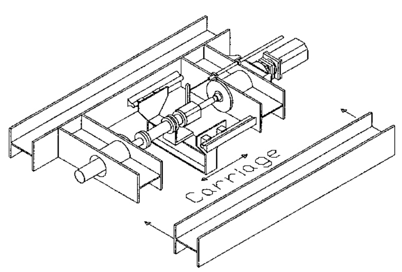

Fig. 1 EMA test stand.

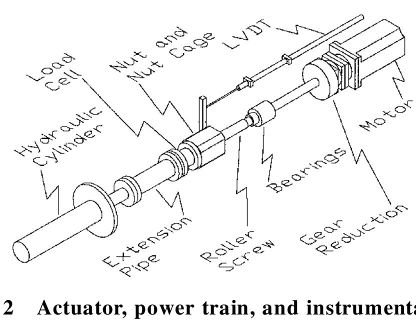

Fig. 2 Actuator, power train, and instrumentation.

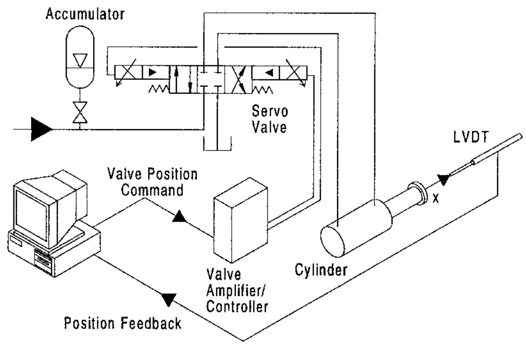

Fig. 3 Hydraulic control loop.

The particular roller screw and nut used are an SKF set with a nonpreloaded nut, a 48-mm nominal-diameter screw, and 20-mm lead. There are six radial/thrust ball bearings that are preloaded with three bearings absorbing compressive loads and three absorbing tensile loads. The gear reducer is a Micron planetary reducer with a 4 to 1 reduction. It was designed to mate directly to the motor used in the system. The motor is a Kollmorgen B-802B servomotor, with a 31.8 ft-lb continuous stall torque and a 95.3 ft-lb peak torque.

The hydraulic system may be used in either a position control mode or in a force control mode. Figure 3 shows the components involved in the position control loop, which was used for these experiments. The valve amplifier/controller contains an analog position controller for the spool of the valve. Using the computer, a digital position control loop for the load is implemented around the valve-spool position loop. The position measurement is derived from the output of a long-stroke LVDT, which is connected to the nut of the actuator.

EMA Models

The parameters were estimated for both a linear and a nonlinear model of the EMA. The difference between these two models is in the treatment of the friction forces in the actuator. It is important to note in the discussion of the nonlinear model that while the equation of motion for the nonlinear model is nonlinear in the state variables, it is linear in the parameters identified. This is an important quality for a least-squares parameter estimation scheme.

Linear Model

A simple rigid body model for the EMA's mechanical system is presented in Fig. 4. The mass of this model M is the total mass of the system reflected to the point of application of the force measured with the load cell. It includes the mass of the nut and the reflected inertia of the roller screw, gear reducer, and motor. The friction is the total friction in the system.

In the linear model the friction force is considered to be viscous (proportional to velocity). This results in a very simple equation of motion for the system:

$$F = M\ddot{x} + B\dot{x} \tag{1}$$

Equation (1) may be considered to represent either a first- or a second-order system, depending on whether velocity or position is used to describe the output of the system. Using the Laplace transform on Eq. (1), two different transfer-function representations of the system may be considered:

$$G_1(s) = \frac{x(s)}{F(s)} = \frac{1}{s(Ms + B)} \tag{1a}$$

$$G_0(s) = \frac{\dot{x}(s)}{F(s)} = \frac{1}{Ms + B} \tag{1b}$$

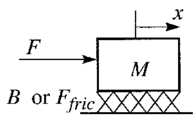

Fig. 4 Rigid body model the EMA.

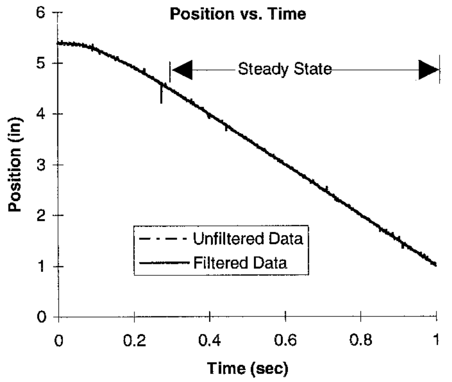

Fig. 5 Position data from a −5 in./s velocity command.

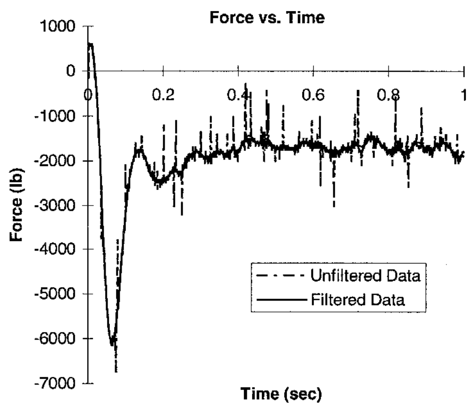

Fig. 6 Force data from a −5 in./s velocity command.

If position is used to describe the output, then it is a second-order, type 1 system, having one pole at zero for a continuous-time model (at 1 for a discrete-time model). If velocity is considered the output, then it is a first-order, type 0 system, with no poles at zero. This is significant because a pole at zero can create difficulties in many parameter estimation procedures. It is therefore desirable to treat this as a first-order system, using velocity as the output. This is possible in the estimation scheme presented in the next section, but is not possible in many other estimation schemes.

Nonlinear Model

To obtain an understanding of the behavior of the friction in the EMA and to obtain a friction model that may be incorporated into the least-squares algorithm, some preliminary experiments were completed. The position plot in Fig. 5 shows the raw and filtered position data obtained from a negative 5 in./s velocity command to the hydraulic controller. The filtered data are difficult to discern in this figure because it is a smooth line tracking through the raw data. The raw and filtered force data corresponding to the position data in Fig. 5 are shown in Fig. 6. Constant-velocity tests were completed for 0.1, 0.2, 0.3, 0.4, 0.5, −0.5, 0.6, 0.7, 0.8, 0.9, 1.0, −1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 5.0, and −5.0 in./s. Above a velocity of 5 in./s, the flow of the hydraulic pump is insufficient to reach the commanded velocity.

The data used to develop the friction model were taken from the end portions of the constant-velocity data, where the velocity has reached steady state. The average force and velocity were found from the steady-state data in each of the constant-velocity tests. Then, these steady-state values were plotted against each other (Fig. 7). Several curve fits were experimented with an attempt to find a mathematical expression for the relationship between the friction force and the velocity. It was found that a logarithmic function resulted in a fairly simple model with a good fit.

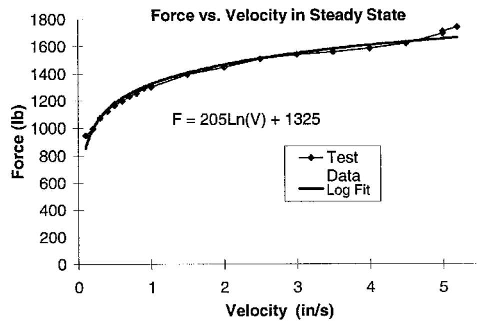

Fig. 7 Relationship between friction force and steady-state velocity.

With the information presented in Fig. 7, a nonlinear, rigid body model of the EMA was developed. This model is presented in Eq. (2):

$$F = M\ddot{x} + F_{\text{fric}} \tag{2}$$

where

$$F_{\text{fric}} = \begin{cases} 0 & \text{for} \quad \dot{x} < 0.001 \text{ in./s} \\ \text{sign}(\dot{x})(b\ \ell n(|\dot{x}|) + c) & \text{for} \quad \dot{x} \geq 0.001 \text{ in./s} \end{cases}$$

$$\text{sign}(\dot{x}) = \begin{cases} 1 & \text{for} \quad \dot{x} \geq 0 \\ -1 & \text{for} \quad \dot{x} < 0 \end{cases}$$

The goal of the identification of a model for the friction was to obtain a fairly simple model that could be used in a least-squares estimation scheme. It should be noted that the logarithmic model is linear in the parameters to be estimated, even though it is not linear in the state variable $\dot{x}$. The fact that it is linear in the parameters allows it to be incorporated into a formulation using a least-squares estimation of the parameters. It might also be noted that this friction model is very similar to the commonly used friction model containing both a static coefficient of friction and viscous coefficient of friction, where the low velocity friction force is dominated by the static term:

$$F_{\text{fric}} = C \text{ sign}(\dot{x}) + B\dot{x} \tag{3}$$

Approach to Estimation

The following is a description of the estimation procedure employed. For the EMA model described by Eq. (1), the results of parameter estimation are M and B; whereas, estimation of the model described by Eq. (2) results in estimates of the coefficients M, b, and c. The differential equations [Eqs. (1) and (2)], hold at any instant in time, including those at which the signals are sampled. If $\ddot{x}$, $\dot{x}$, and F are known for m different times, then we could write m equations and use them to find the n unknown parameters. The difficulty with this is that we cannot conveniently compute $\ddot{x}$ and $\dot{x}$, since this involves differentiation of the sampled variable x. In practice, the differentiation of a sensor output to find the derivative of a variable is often inaccurate, since sensor signals almost always contain noise that is amplified by the differentiation process. A solution to this problem is to filter the signals before calculating the derivatives. Mathematically speaking, this is perfectly valid for a linear system if duplicate filters are used on both the input and the output. Consider the transfer function representation of a linear system, where the differential equation describing the system is represented as a ratio of polynomials of the differential operator s. The coefficients of the polynomials of $G(s)$, the transfer function, are the unknown parameters to be identified. This representation is shown in Fig. 8 with duplicate filters operating on the system input F, and the system output x. The transfer function and, therefore, the differential equation describing the relationship between the system input and output, can also be used to describe the relationship between the filtered input and filtered output.

The complete process used for parameter estimation is depicted in Fig. 9. The EMA system is excited using the hydraulic test stand under velocity control. The input to the system, the force exerted on the nut, is an artifact of the velocity control loop, the complete hydraulic-EMA system dynamics, and velocity command. The input and output are filtered with duplicate numerical filters. The filtered output is numerically differentiated twice using a second-order, central-difference method to obtain velocity and acceleration. A least-squares algorithm is used for parameter estimation after processing of the data.

A zero-phase filter is used on both the raw force and position data. Although the phase shift created by the filter should not ideally affect the estimation of the parameters for the linear model, a zero-phase filter performed better in the estimation of the nonlinear model. A 12-pole, zero-phase filter is obtained by using a 6-pole Butterworth filter. The data are passed through first in the forward direction and then again in the reverse direction, resulting in twice the attenuation and zero phase shift. Choosing the cutoff frequency for this filter involved a few iterations. The final value used was 25 Hz.

In the literature on the estimation of continuous-time linear systems, there are several solutions presented for the design of the filters. They are called state-variable filters by a few authors. Bai,[6] Zhao et al.,[7] and Young[4] all state the best performance is obtained in the estimation procedure if the frequency band of the filters matches that of the system being estimated. Bai[6] and Young[4] propose that the filter should be found recursively, using the denominator from the transfer function of previously estimated models. However, no one addresses the complicating issues involved with a type 1 system like that estimated here. If the denominator of the type 1 linear system was used for the filter, then the filter would be marginally stable because of the free integrator [the free s in the denominator polynomial of $G_1(s)$]. There would, therefore, be a nondecaying difference between the measured and the filtered data. In this case, convergence to the correct parameters is very questionable. If the free integrator is eliminated from the filter,

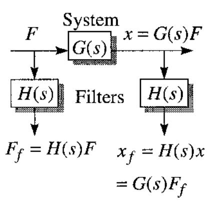

Fig. 8 Linear system with filtered input and output.

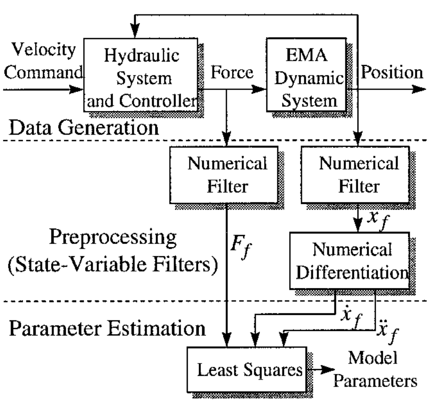

Fig. 9 Parameter estimation scheme.

then there is considerable ambiguity in defining a match between the frequency bands of the filters and the system. The method used here was to use a filter with a cutoff frequency slightly greater than the breakpoint of the first-order EMA system. The effects of these filters on the force and position data can be seen in Figs. 5 and 6.

The estimation scheme described earlier is implemented as a batch process. The data are collected and then processed as a whole. With only slight modification it could be implemented recursively, operating on the data as they are collected. The zero-phase filters could not be implemented in this case, and standard filters would have to be used.

Least-Squares Estimation

Least-squares is utilized in the system identification procedures described here. Least-squares may be applied to any set of equations that can be put into the vector-matrix regression-model form:

$$\boldsymbol{Y}_{(m\times 1)} = \Phi_{(m\times n)} u_{(n\times 1)} + \boldsymbol{E}_{(m\times 1)}, \qquad m > n \tag{4}$$

In Eq. (4), the vector $\boldsymbol{Y}$ and the matrix Φ are known, and the vector u is to be determined. The vector $\boldsymbol{E}$ is a sequence $\{e(i), i = 1, 2, \ldots, m\}$ of independent random variables with zero mean, and is independent of Φ. For the parameter estimation problem, the elements of the vector $\boldsymbol{Y}$ and the matrix Φ in Eq. (4) must be either measured variables or functions that can be calculated from the measured variables. u is the vector of parameters to be determined in the least-squares sense. Φ is a matrix formed using the regressors. Specifically, in the estimation problem, the regressors are those functions that are multiplied by the parameters in the differential equations. There are no restrictions in linear-regression that the regressors be linear functions of the measured data, only that the equations are linear in the parameters to be determined.

Equation (4) is the standard linear-regression model that may be solved using the left Φ^*:

$$\Phi^* \equiv (\Phi^T\Phi)^{-1}\Phi^T \tag{5}$$

The least-squares solution is found by multiplying the pseudoinverse by the known vector:

$$\hat{u} = \Phi^* Y \tag{6}$$

If $\Phi^T\Phi$ is nonsingular, then the pseudoinverse solution for the parameters, $\hat{u}$, is the unique solution to the following least-squares condition:

$$\min_{\hat{\theta}} \| Y - \Phi\hat{u} \|^2 \tag{7}$$

where $\| \ \|$ denotes the Euclidean norm.

The differential equations in both Eqs. (1) and (2) are linear in the parameters. Therefore, a vector–matrix equation in the form of Eq. (4) may be found by writing the differential equation at the instants in time when the force and the derivatives of x are known from experimental measurement. For the linear model, the elements of Eq. (4) are

$$\boldsymbol{Y} = \boldsymbol{F}_f = \begin{Bmatrix} F_f(1) \\ F_f(2) \\ \vdots \\ F_f(i) \\ \vdots \\ F_f(m) \end{Bmatrix} \tag{8a}$$

$$\Phi = \begin{Bmatrix} \ddot{x}_f(1) & \dot{x}_f(1) \\ \ddot{x}_f(2) & \dot{x}_f(2) \\ \vdots & \vdots \\ \ddot{x}_f(i) & \dot{x}_f(i) \\ \vdots & \vdots \\ \ddot{x}_f(m) & \dot{x}_f(m) \end{Bmatrix} \tag{8b}$$

$$u = \begin{Bmatrix} M \\ B \end{Bmatrix} \tag{8c}$$

For the nonlinear model, the elements of Eq. (4) are

$$\boldsymbol{Y} = \boldsymbol{F}_f = \begin{Bmatrix} F_f(1) \\ F_f(2) \\ \vdots \\ F_f(i) \\ \vdots \\ F_f(m) \end{Bmatrix} \tag{9a}$$

$$\Phi = \begin{Bmatrix} \ddot{x}_f(1) & \text{sign}[\dot{x}_f(1)]\ell n[|\dot{x}_f(1)|] & \text{sign}[\dot{x}_f(1)] \\ \ddot{x}_f(2) & \text{sign}[\dot{x}_f(2)]\ell n[|\dot{x}_f(2)|] & \text{sign}[\dot{x}_f(2)] \\ \vdots & \vdots & \vdots \\ \ddot{x}_f(i) & \text{sign}[\dot{x}_f(i)]\ell n[|\dot{x}_f(i)|] & \text{sign}[\dot{x}_f(i)] \\ \vdots & \vdots & \vdots \\ \ddot{x}_f(m) & \text{sign}[\dot{x}_f(m)]\ell n[|\dot{x}_f(m)|] & \text{sign}[\dot{x}_f(m)] \end{Bmatrix} \tag{9b}$$

$$u = \begin{Bmatrix} M \\ b \\ c \end{Bmatrix} \tag{9c}$$

In Eqs. (8) and (9), each row of $\boldsymbol{F}_f$ and Φ correspond to one sample time at $t = t_i$, and m rows are formed for the m samples that are used in the estimation procedure. The least-squares solution of Eqs. (8) and (9) will minimize the 2-norm size for the vector of errors, $\{\boldsymbol{F}_f - \Phi\hat{u}\}$. In other words, the least-squares solution for the model parameters minimizes the size of the vector of errors between the measured force and the force calculated using the estimated parameters and the regressors.

Other Estimation Methods

The parameter estimation method described earlier is applicable to continuous-time, linear, and nonlinear differential equation models. The only restriction is that the model be linear in the parameters. The minimization scheme, least-squares, is simple and robust. Admittedly, because of the use of the data filters, it would be impossible to completely validate the approach for the nonlinear model. However, it will be seen in the next section that the results for the specific case to which it was applied are encouraging.

The most common estimation methods use discrete-time models. While these methods are perhaps the simplest to use, their application is restrictive in the types of systems to which they apply. Discrete-time models are in general not restricted to have regressors that are a linear function of the measured outputs. However, it is difficult to describe most nonlinearities in discrete-time. Nonlinearities are usually best described in continuous-time, and are not transformed to and from discrete-time as easily as linear models, such as transfer functions.

With discrete-time models, the usefulness of physical insight for a system is diminished, complicating the estimation procedure. The transformation of the linear continuous-time model, Eq. (1), to discrete-time would result in a type 1 system and up to six parameters, depending on the transformation used. The type 1 system results because the discrete model is a function of the actual measured output position, rather than the velocity. In continuous-time, the model may easily be simplified to a type 0 system because velocity can be considered to be the output. The filtered velocity is a direct result of the state-variable filters. A type 1 system may be difficult to estimate because it is marginally stable, and the resulting model may very well be an unstable one. The additional parameters of the discrete-time system also create difficulties. There will be a larger variance for the identified parameters, and a greater possibility of finding a good match for the data used in estimation that does not apply to other data. Furthermore, transformation of the estimated model back to continuous-time is not a trivial manner. As discussed by Sinha,[8] the choice of transformations is dependent on many assumptions and/or properties of the model. It is very likely that in the transformation of the identified parameters back to continuous-time, it would be impossible to simplify the model to one containing only two parameters. In short, much utility and intuition are lost in the use of discrete-time models when the system is fairly well understood in continuous-time.

It can easily be shown that if $\boldsymbol{E}$ is as described for Eq. (4), then the mathematical expectation of the estimated parameter vector $\hat{u}$, given by Eq. (6) is the true parameter vector u. An estimate with this property is called unbiased. If, however, the conditions given for $\boldsymbol{E}$ are violated, then the estimate will be biased; it will not necessarily converge to the true parameters. In many practical cases, the bias may be insignificant compared to other errors. If the source of the error vector $\boldsymbol{E}$, is independent, zero mean sensor noise on both the force and position measurement, then the scheme in Fig. 9 will obviously violate the conditions set forth for $\boldsymbol{E}$. One reason for this is because the noise is passed through the filters along with the data. An independent sequence will not be independent after it has passed through a nonstationary process, such as the filters.

One possible solution to the bias problem might be to use instrumental-variable (IV) and/or bootstrap methods for the estimation algorithm. However, in the estimation scheme described by Fig. 9, the data are generated under closed-loop control. While this does not completely eliminate the possibility of using IV and bootstrap methods for the estimation algorithm, it does make their implementation difficult and their utility questionable. Examples of these methods are presented by Young and Jakeman[9] and by Stoica and Soderstrom.[10] These methods, which use a minimization scheme similar to least-squares, are used mainly as a solution to this classical error in variables problem associated with least-squares. However, the utility and robustness of the linear-regression model and least-squares estimation may very well offset the possibility of a small error being introduced into the estimation. The IV and bootstrap methods utilize another variable that is independent of the prediction error, the instrument. This variable is usually generated using the input to the system, which is assumed to be independent of any error in the output measurement. In closed-loop control, this is no longer true, and more sophisticated methods must be used in the generation of the instrumental-variable. In this case, the costs of IV and bootstrap methods probably outweigh the benefits.

Another possible solution to the bias problem is to formulate the minimization problem to minimize a cost function that is the difference between the output of a predictor and the actual output of the system. The predictor should be determined using a stochastic analysis of the system. It is a function of the data and the model parameters, and gives the statistically expected output of the system. This is a general statement of the methods used in stochastic parameter estimation and includes many of the methods used. It is the underlying theme of Ljung's classic text on system identification.[1] If the stochastic analysis results in a linear-regression model, then the minimization can be solved analytically using least-squares. In general, however, the analysis will lead to a cost function that must be minimized using search algorithms such as steepest descent or Gauss–Newton methods. These methods suffer from the usual complications of iterative search schemes such as local minima. They also suffer from other complications like predictor stability, which is a function of the parameters that are changing during the search, increased numbers of parameters to be identified, and difficulties in computing the gradient functions. Additional parameters often result from the need to simultaneously estimate parameters for the noise model developed in the stochastic analysis. Several software packages are available that implement black box models and different search schemes, including a fairly robust package developed by Ljung.[11] However, these packages do little in terms of continuous-time estimation. There are few examples of such methods applied to continuous-time models in the technical literature. Some of these, such as a method described by Young and Jakeman,[9] can be considered stochastic, sampled data extensions of early deterministic, analog work by Kreisselmeier.[12]

Estimation of the EMA Models

Four different sets of data were used in the parameter estimation and model validation. They were all generated under closed-loop velocity control (Fig. 9). Figures 10–13 show this data after processing by the filters in Fig. 9. The data shown in Fig. 12 were used to estimate the parameters, and the other data were used in model validation.

The excitation signal, or input, used to generate the data set must be persistent to obtain good results. Obviously, data collected from a zero input would not be very useful in estimation. It follows from the condition for uniqueness of the least-squares solution, that the minimum is unique if $\Phi^T\Phi$ has full rank. This is called the excitation condition, and the input is persistently exciting if it is satisfied. In some practical situations, it is possible to generate a series of nonperiodic steps, or a close approximation of one, as input to the system. Such an input will be persistently exciting in nearly any situation. In this case, it is impossible to generate a step input. In practical situations, the boundary between a persistent input and a nonpersistent signal is not important, since the convergence properties of the estimation scheme will probably render it useless at levels of excitation far from the level of one on this boundary. It is wise to avoid inputs that are constant for long periods of time and/or that are composed of a very few sinusoidal frequencies. For example, the data sets in Fig. 10 have questionable application in estimation. They are, therefore, only used in verification.

The estimation procedure described previously was used in the estimation of the parameters for both the linear and nonlinear models of the EMA. The results of the estimation are given in Table 1. The data were estimated using the experimental data presented in Fig. 12. As would be expected, these data are the best match for the models in Table 1, as seen in the following section.

Table 1 Estimated parameters

Linear model		Nonlinear model		
M	*B*	*M*	*b*	*c*
60 lb s²/in.	580 lb s/in.	58 lb s²/in.	502 lb	1354 lb

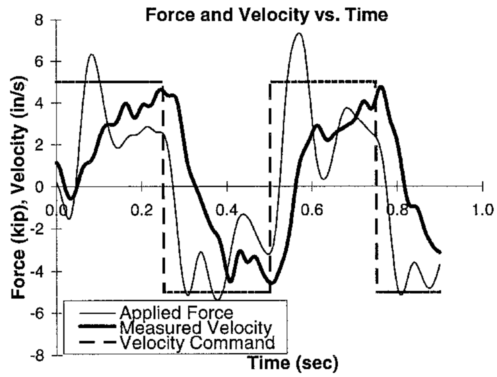

Fig. 12 Data from a 2-Hz square wave command.

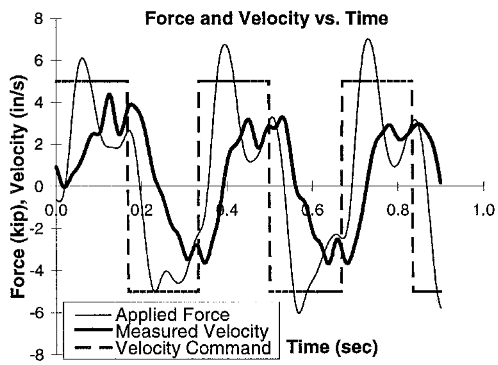

Fig. 13 Data from a 3-Hz square wave command.

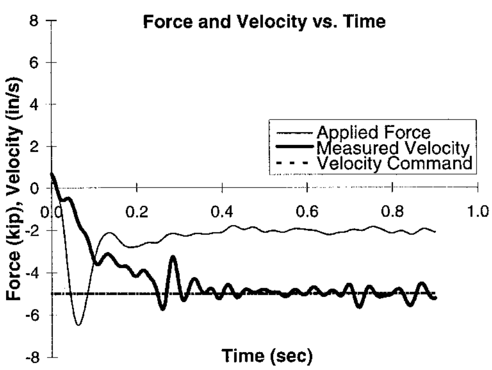

Fig. 10 Data from a step command.

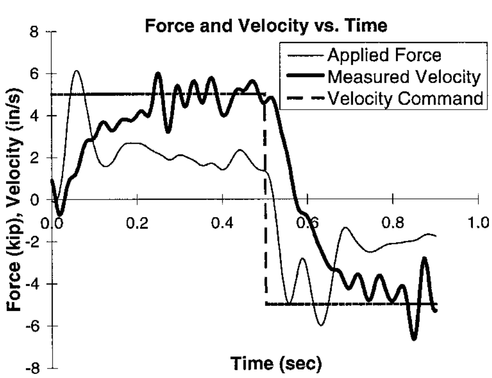

Fig. 11 Data from a 1-Hz square wave command.

Model Validation

The models were verified by simulating the models using the measured force data as the input to the systems. This was done with the force data for Figs. 10–13, and the corresponding results are shown in Figs. 14–17. In each of these figures, the simulated velocity is shown for both the linear and nonlinear models along with the measured velocity.

From the previous four figures (Figs. 14–17), it can be seen that both models follow the rigid body trajectories of the actual system fairly well. Neither, however, capture the higher-frequency oscillations that are probably the effects of unmodeled disturbances exciting higher-order dynamics in the actual system. A more accurate model of the actual system might distribute the total mass of the system into masses separated by a spring. The spring would represent torsional windup of the roller screw. In Figs. 14 and 16, it is obvious that the nonlinear

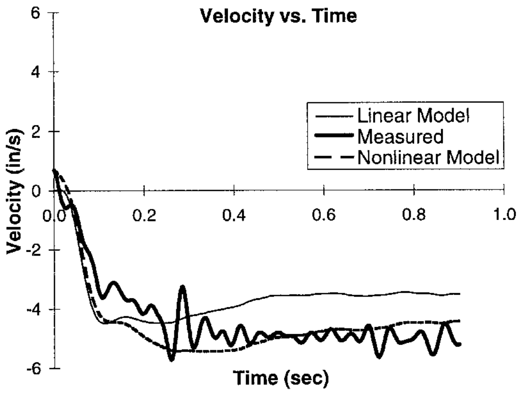

Fig. 14 Simulation using the step force data as input.

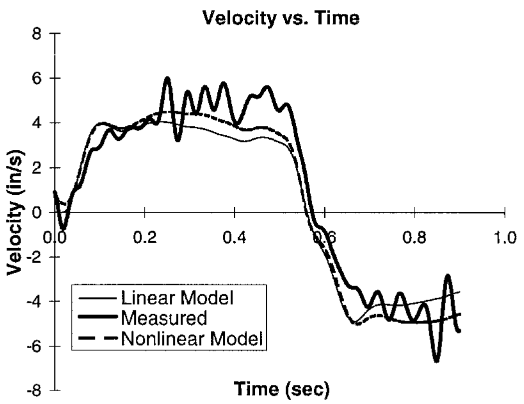

Fig. 15 Simulation using the 1-Hz force data as input.

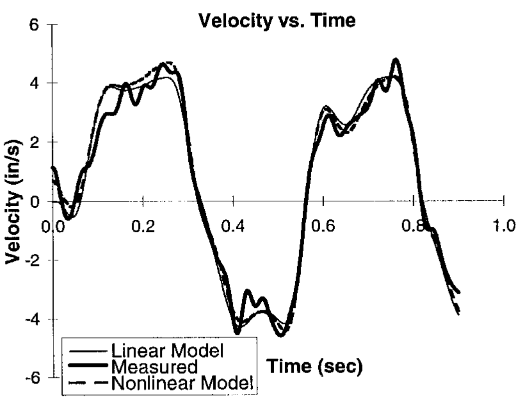

Fig. 16 Simulation using the 2-Hz force data as input.

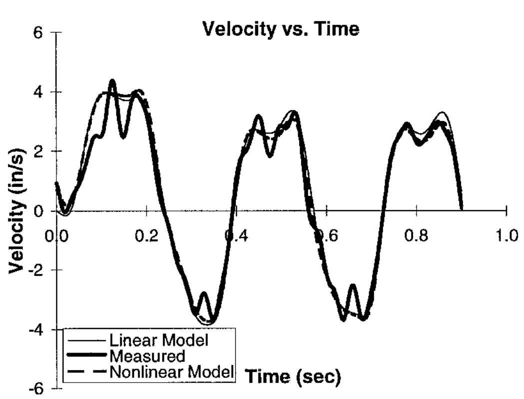

Fig. 17 Simulation using the 3-Hz force data as input.

model is a better predictor of the steady-state performance of the actual system. This is because of the insignificance of the mass in steady state and the dominance of the nonlinear friction forces.

Conclusions

An approach to the identification of type 1, continuous-time, linear, and nonlinear models of dynamic systems has been developed and presented. A type 1 system, which has a free integrator in the continuous-time transfer function, is very common for electromechanical actuators used in aerospace systems and also for electromechanical actuation of machine tool axes. Therefore, the extent of potential applications for the approach is very large. The approach reduces the order of the model by eliminating the free integrator to obtain a type 0 system. This is done by considering the output to be velocity rather than position. The parameter estimation scheme that is used in the approach is consistent with this reduction of the system order.

The estimation scheme directly estimates the parameters of the continuous-time model using data that are obtained from experiments and filtered using state-variable filters. This is important because the model-order reduction is not possible using the more popular discrete-time estimation schemes. Furthermore, the incorporation of certain types of empirically developed nonlinearities is possible within this scheme. Although it is difficult to prove that the estimated parameters would converge to the true parameters of the nonlinear system if they existed, it is possible to incorporate them into the formulation of the estimation scheme, and the model resulting from doing so in the application in this paper is a good predictor of the experimental data. It is very difficult, if not impossible, to incorporate these types of nonlinearities into a discrete-time estimation scheme, since they are developed from continuous-time relationships and their transformation into discrete time is probably not known.

The estimation scheme uses least-squares approach in the computation of the unknown parameters. This is a pragmatic solution to the estimation problem. The rationale for the use of the simple least-squares algorithm rather than more complicated methods such as instrumental-variable methods, which may be more applicable in theory, is that these more complicated methods suffer from many practical problems that may affect the estimated parameters even more than the potential bias introduced by using least-squares.

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